AS1300 - Tutorial 1

Solutions

1. a) contents of a closed flask
   - system (mass remains constant)

   b) air inside a tyre being inflated
   - neither (both mass and volume vary)

   c) sand in a sand clock
   - system.

   d) Air conditioner
   - control volume (involves the flow of air)

   e) a spring in a machine
   - system.

   f) filling of a gas cylinder
   - control volume.

   g) water flowing through a shower head
   - control volume

   h) lungs of a living human
   - neither.

   i) combustion inside a closed vessel
   - system

2. Intensive property $\Rightarrow$ independent of size / amount of substance

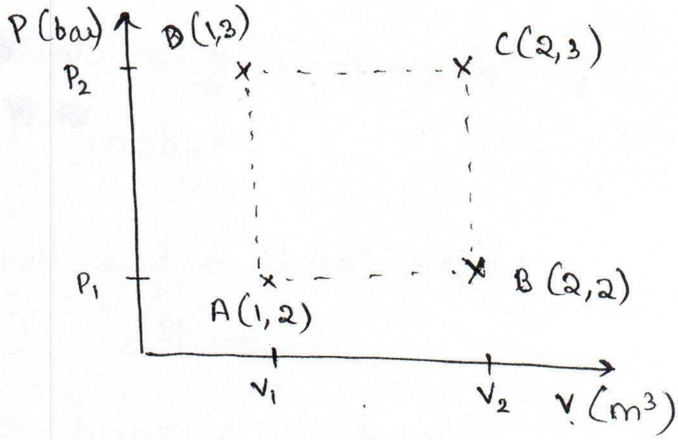
   (a) pressure, (b) Temperature, (f) P/T.

   Extensive property

   (c) volume, (d) $E+PV$, (e) PT/V (f) PV/T^2

3. If the integral evaluated through different paths such as A-B-C, A-D-C and A-C gives the same value, then that integral can be considered path independent.

i) $\int_1^3 p\,dV$



$\Rightarrow A \rightarrow B = 2\,bar \times 1\,m^3$

$= 200\,kJ$

$B \rightarrow C \Rightarrow dV = 0$

$\Rightarrow \int p\,dV \Rightarrow 0$

$\Rightarrow$ for A-B-C $\int_1^2 p\,dV = \underline{\underline{200\,kJ}}$

for A-D-C

$A \rightarrow D \quad \int p\,dV = 0$

$D - C \quad \int p\,dV = 3\,bar \times 1\,m^3 = 300\,kJ.$

$\Rightarrow$ for A-D-C $\int_1^3 p\,dV = \underline{\underline{300\,kJ}}.$

for A-C,

p is a linear function of V

area under the line AC is $\int p\,dV$.

area of ($V_1 A C V_2$) trapezium

$= \frac{1}{2} h (b_1 + b_2)$

$= \frac{1}{2} \times (1\,m^3)(2\,bar + 3\,bar)$

$= \underline{\underline{250\,kJ}}$.

All three are different.

Hence <u>path dependent</u> property.

ii) $\int_1^3 V\,dp$.

For path A-B-C   for A-B $\int V\,dp = 0$.

for B - C

$$\int V dp = (2m^3)\, 1\ \text{bar} = 200\ kJ$$

$\Rightarrow$ A-B-C $= \underline{200\ kJ}$

for A-D-C

A-D

$$\int V dp = (1m^3)\, 1\ \text{bar} = 100\ kJ$$

D-C

$$\int V dp = 0$$

$\Rightarrow$ A-D-C $= \underline{100\ kJ}$.

for A-C.

$$\int V dp = \frac{1}{2} \times (1\ \text{bar}) \times (2m^3 + 1m^3) = \underline{150\ kJ}.$$

All three are different.

Hence path dependent.

iii) $\int p dV + \int V dp$.

for A-B-C

$$\int p dV + V dp = \underbrace{200\ kJ}_{\text{from (i)}} + \underbrace{200\ kJ}_{\text{from (ii)}} = \underline{400\ kJ}.$$

Similarly for A-D-C and A-C, the value remains $\underline{400\ kJ}$

Hence path independent

iv) $\int p dV - V dp$.

A-B-C	A-D-C	A-C
$= \underline{0\ kJ}$	$= 300 - 100$	$= 250 - 150$
	$= \underline{200\ kJ}$	$= \underline{100\ kJ}$

$\Rightarrow$ path dependent.

4. Given $\quad \delta\phi = f(T)\,dT + \frac{RT}{V}\,dV$.

If ϕ is an exact differential, it is a property. ie.

ϕ is a function of T and V.

Hence.

$$d\phi = \left(\frac{\partial\phi}{\partial T}\right)dT + \left(\frac{\partial\phi}{\partial V}\right)dV.$$

Say

$$= M\,dT + N\,dV.$$

ϕ being an exact diffential would imply.

$$\frac{\partial^2\phi}{\partial T\,\partial V} = \frac{\partial^2\phi}{\partial V\,\partial T}$$

$$\Rightarrow \frac{\partial M}{\partial V} = \frac{\partial N}{\partial T}$$

In this case.

$$M = f(T) \Rightarrow \frac{\partial M}{\partial V} = 0$$

and

$$N = \frac{RT}{V} \Rightarrow \frac{\partial N}{\partial T} = R/V$$

$$\Rightarrow \frac{\partial M}{\partial V} \neq \frac{\partial N}{\partial T}$$

Hence ϕ is not a property.

5. Initial pressure in ~~the~~ either side of the tube $= P_{atm}$.

Initial volume $= A \times 45 \times 10^{-2}\ m^3$

Initial PV $= P_{atm} \times A \times 45 \times 10^{-2}$

After becoming vertical, say mercury descended by an amount of h cm.

$\Rightarrow$ length on top $= 45 + h$.

volume on top $= A \times (45 + h) \times 10^{-2}\ m^3$

Let the pressure on the top be P_{top}.

Using $PV = \text{constant}$

we have,

$$P_{atm} \times A \times 45 \times 10^{-2} = P_{top} \times A \times (45+h) \times 10^{-2}$$

$$\Rightarrow P_{top} = P_{atm} \times \frac{45}{45+h} \quad \text{——— (1)}$$

Similarly for the bottom part,

let the pressure be P_{bottom}

and volume being $A \times (45-h) \times 10^{-2}\ m^3$

and using $PV = \text{constant}$

we get

$$P_{atm} \times A \times 45 \times 10^{-2} = P_{bottom} \times A \times (45-h) \times 10^{-2}$$

$$\Rightarrow P_{bottom} = P_{atm} \times \frac{45}{45-h} \quad \text{——— (2)}$$

From hydrodynamics, we know that

$$P_{bottom} = P_{top} + \rho_{Hg}\, g \times h_{gh} \quad \text{——— (3)}$$

and

$$P_{atm} = \rho_{Hg} \times g \times 0.76 \longrightarrow \text{known.}$$

Using (1) - (3), we get.

$$\rho_{Hg} \times g \times 0.76 \times \frac{45}{45-h} = \rho_{Hg} \times g \times \frac{0.76 \times 45}{45+h} + \rho_{Hg} \times g \times \cancel{0.01}\ 10 \times 10^{-2}$$

$$\cancel{0.76} \quad \frac{1}{45-h} = \frac{1}{45+h} + \frac{1}{7.6 \times 45}$$

solving, $h^2 + 684h - 2025 = 0 \quad \Rightarrow h = 2.948, -686.95$

$$\Rightarrow \boxed{h = 2.948\ cm} \text{ //.}$$

TEMPERATURE

6.

$$R = R_0[1 + \alpha(T - T_0)]$$

$$R - R_0 = \alpha R_0 (T - T_0)$$

$$\frac{R - R_0}{T - T_0} = \alpha R_0 \quad \text{(Constant)}$$

For $\cancel{R}$ $T = 0\,^\circ C$ $\qquad R_0 = 51.39\,\Omega$

$T = 91\,^\circ C$ $\qquad R_{91} = 51.72\,\Omega$

To find: R_{50} for $T = 50\,^\circ C$

By Calibration, we have

$$\frac{R_{50} - R_0}{R_{91} - R_0} = \frac{50 - T_0}{91 - T_0}$$

$$R_{50} - R_0 = (R_{91} - R_0)\left(\frac{50 - T_0}{91 - T_0}\right)$$

$$R_{50} = R_0 + (R_{91} - R_0)\left(\frac{50 - T_0}{91 - T_0}\right)$$

$$= 51.39 + (51.72 - 51.39)\left(\frac{50 - 0}{91 - 0}\right)$$

$$\boxed{R_{50} = 51.57\,\Omega}$$

7.

$$T_A = p + q T_B + r T_B^2$$

Given: when $T_A = 0$, $T_B = 0$

i.e $0 = p + q(0) + r(0)^2$

$\Rightarrow p = 0$

Also when $T_A = 100$, $T_B = 100$

i.e $100 = 0 + q(100) + r(100)^2$

$1 = q + 100r$ — ①

Also when $T_A = 51$, $T_B = 50$

i.e $51 = 0 + q(50) + r(50)^2$

$\frac{51}{50} = q + 50r$ — ②

Solving ① & ② we get

$r = -0.0004$

$q = 1.04$

Thus $T_A = 1.04\, T_B - 0.0004 T_B^2$

For $T_B = 30°C$

$T_A = 1.04 \times 30 - 0.0004 \times (30)^2$

$\boxed{T_A = 30.84°C}$

At 0°C, pointer on the Al rod reads 0°C

At 100°C, expansion of Al rod is equal to the total length of the scale.

$$\therefore \quad \Delta L_{AL,100} = L_{scale,100}$$

$$0.2221 \times 10^{-4} \times 100 \times L_{AL,0} = (1 + 10^{-5} \times 100) \times L_{scale,0}$$

$$\frac{L_{AL,0}}{L_{scale,0}} = \frac{(1 + 10^{-5} \times 100)}{0.2221 \times 10^{-4} \times 100} = 450.7$$

At any temp T, the pointer is in range $0 \le T \le 10$

$$T = 100 \times \Delta L_{AL,Actual\_T} / (L_{scale,0} + \Delta L_{scale,Actual\_T})$$

$$= \frac{100 \times 0.2221 \times 10^{-4} \times T_{Actual} \times L_{AL,0}}{\{(1 + 10^{-5} \times T_{Actual}) \times L_{scale,0}\}}$$

Using the values of $\frac{L_{AL,0}}{L_{scale,0}}$ and $T = 40°$

$$40 = \frac{100 \times 0.2221 \times 10^{-4} \times T_{Actual} \times 450.7}{(1 + 10^{-5} \times T_{Actual})}$$

$$T_{Actual} = \frac{40}{(0.2221 \times 10^{-2} \times 450.7 - 40 \times 10^{-5})}$$

$$\boxed{T_{Actual} = 39.97°C}$$

9. When the valve is opened enough, argon flows into B and exerts a pressure of 150 kPa on the piston, and the piston starts to move. The volume (total) starts to increase and the pressure starts falling down. When the pressure falls below 150 kPa, the piston will stop ~~flow~~ moving. Thus, the final pressure in the system will be 150 kPa.

Taking A & B as a system:-

Initial pressure $P_1 = 250\,kPa$

Initial volume $V_1 = 0.4\,m^3$

Final pressure $P_2 = 150\,kPa$, Final Volume $= V_2$ n

Since $PV =$ constant,

$$250 \times 0.4 = 150 \times V_2$$

$$V_2 = \frac{\overset{5}{\cancel{250}} \times 0.4}{\underset{3}{\cancel{150}}} = \frac{2}{3}\,m^3$$

• Work done by the argon gas $= P(V_2 - V_1)$

$$= 150 \times \left(\frac{2}{3} - 0.4\right) = \cancel{180 \times \frac{10}{3}}$$

$$= 150 \times \left(\frac{2}{3} - \frac{4}{10}\right) = 150 \times \frac{8}{30} = 40\,kJ$$

• Work interaction for the atmosphere.

$$W_{atm} = -P_{atm} \times (V_2 - V_1)$$

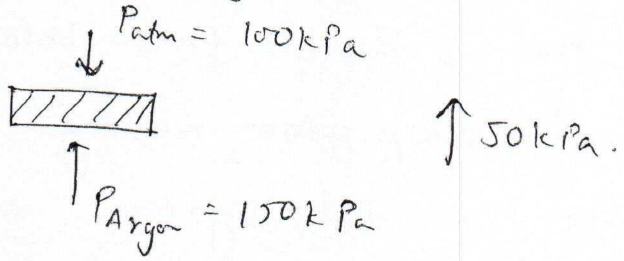
[negative sign because the piston is worki on the atmosphere]

$$= -100 \times \left(\frac{2}{3} - 0.4\right)$$

$$= -10\cancel{0} \times \frac{8}{3\cancel{0}} = -26.67\,kJ$$

• Work interaction for the piston.

Since 150 kPa is needed to lift the piston which includes 100 kPa of atmospheric pressure, the total pressure thus exerted ~~by~~ on the piston is 50 kPa.



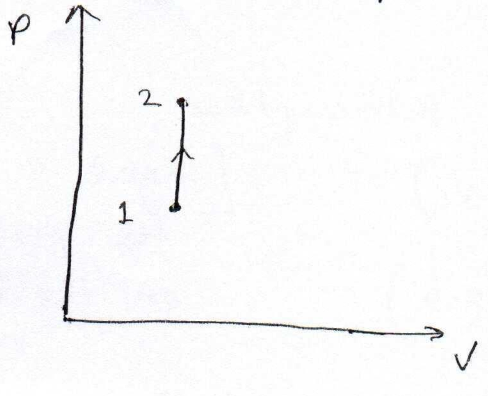
Thus, work interaction for the piston = work done on the piston

$$= -\left[150-100\right] \times \left(\frac{2}{3} - 0.4\right)$$

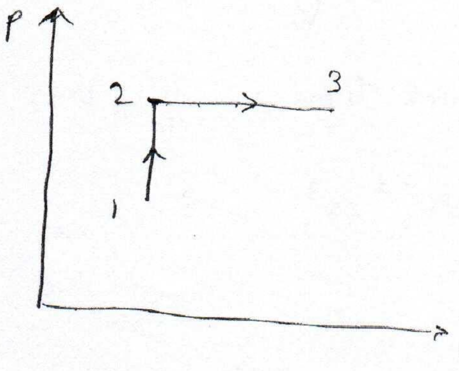
$$= -50 \times \frac{8}{30} = -13.33 \text{ kJ}$$

(negative sign because resultant work is being done on the piston).

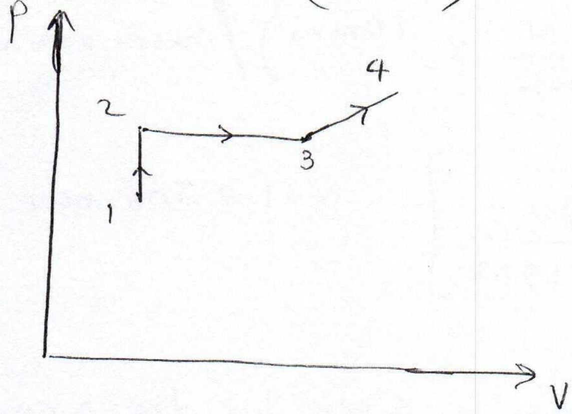
10. A pressure of 1.2 bar is required to lift the piston. Since the initial pressure is 1 bar, a constant volume heating process will initially occur until pressure increases to 1.2 bar. On a PV diagram, this can be represented as



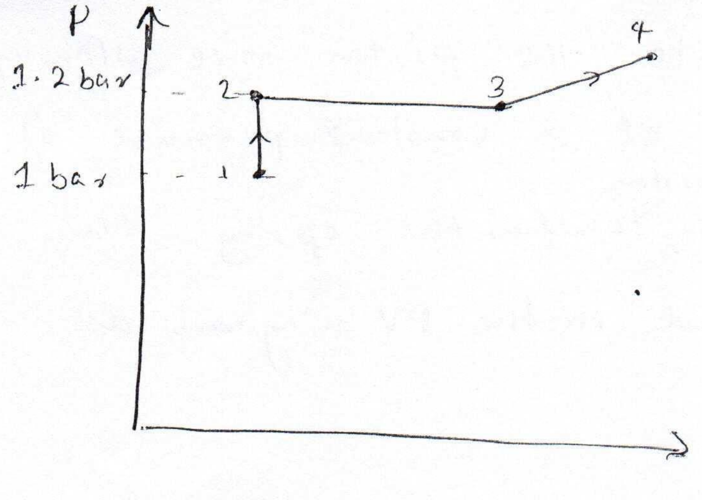
Further heating would make the piston rise slowly, thus increasing the volume at a constant pressure of 1·2 bar, until the ~~pressure~~ piston touches the spring. This process can be represented in the PV diagram as process 2-3:-



With continued heating, as the pressure rises up, the piston keeps on compressing the spring. Since the force exerted by the spring is proportional to the piston's displacement and hence the volume swept by the piston, the gas pressure increases linearly with volume (3-4)



The heating is finally stopped when the spring is compressed by 1 cm (4 on the PV diagram).



Area of the piston's cross-section $= \frac{\pi}{4} \times 0.15^2 \approx 0.01767 m^2$

$V_1 = A \times 0.25 m \approx 4.417 \times 10^{-3} m^3$

$V_1 = V_2$

$V_3 = A \times (0.25 + 0.10) \approx 6.18 \times 10^{-3} m^3$

$V_4 = A \times (0.25 + 0.10 + 0.01) \approx 6.36 \times 10^{-3} m^3$

volumes.

$P_1 = 1$ bar

$P_2 = 1.2$ bar

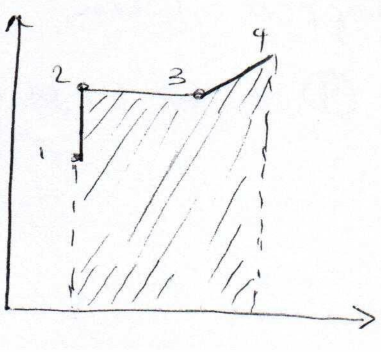
$P_3 = 1.2$ bar

$P_4 = (1.2 \times 10^5) + \left[\left(10 \frac{N}{mm} \times 10 mm\right) / \text{cross-section area}\right]$

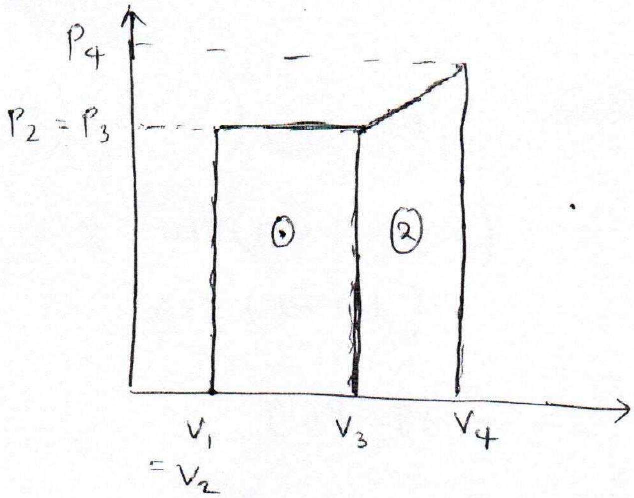
$= 1.2 \times 10^5 + \left[\frac{100}{0.01767}\right] \approx 1.2566$ bar

Pressures

- Work interaction for gas is given by the area under the curve.



The area under the curve can be evaluated as follows:-



① – rectangle

② – trapezoid

$$Area_1 = (V_3 - V_1) \times P_2 = (6.18 - 4.417) \times 10^{-3} \times 1.2 \times 10^5$$

$$= 211.56 \text{ J}$$

$$Area_2 = \frac{1}{2}(P_3 + P_4)(V_4 - V_3)$$

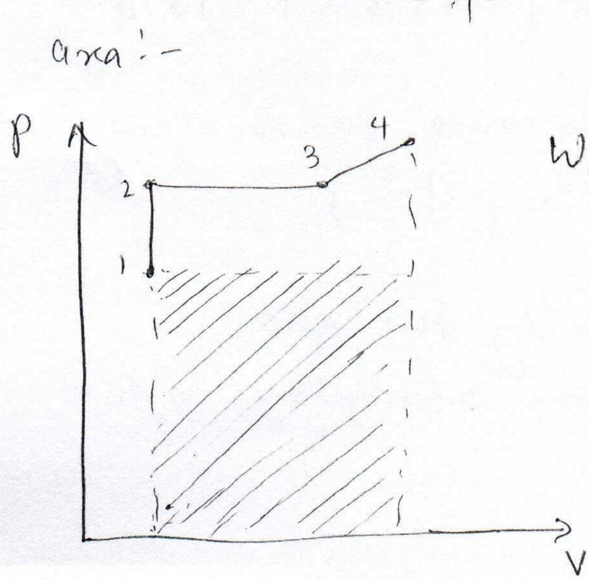
$$= \frac{1}{2}(1.2 + 1.2566) \times 10^5 \times (6.36 - 6.18) \times 10^{-3}$$

$$= 22.1094 \text{ J}$$

$\therefore$ Total area = Total work done by the gas

$$= 211.56 + 22.1094 = 233.6694 \text{ J}$$

- Work interactions for atmosphere is given by the following area:-



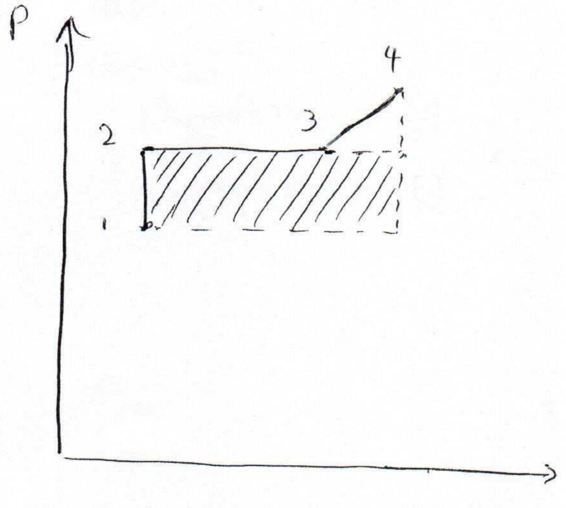
$$W_{atm} = -(V_4 - V_1) \times P_1$$

$$= -(6.36 - 4.417) \times 10^{-3} \times 10^5$$

$$= -194.3 \text{ J}$$

(negative sign because gas is doing work on the atmosphere).

• Work interaction for piston :-



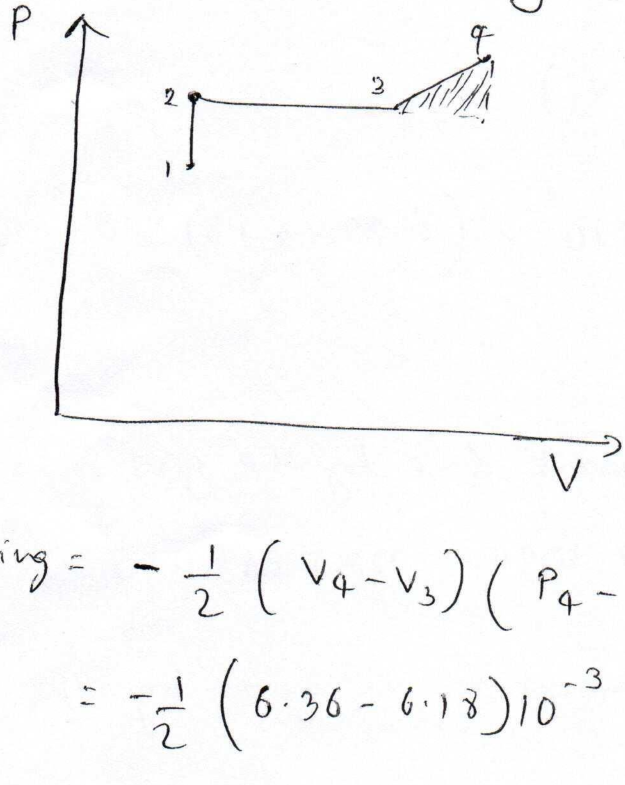
$$W_{piston} = -(V_4 - V_1)(P_2 - P_1)$$

$$= -(6.36 - 4.417) \times 10^{-3} \times (1.2 - 1) \times 10^5$$

$$= -38.86\,J$$

(negative sign because gas is doing work on piston).

• Work interaction for spring :-



$$W_{spring} = -\frac{1}{2}(V_4 - V_3)(P_4 - P_3)$$

$$= -\frac{1}{2}(6.36 - 6.18)10^{-3} \times (1.2566 - 1.2) \times 10^5$$

$$= -0.5094\,J$$ (negative because gas is doing work on piston)

Note that the total work done by the gas is equal to the work done on the atmosphere, piston and spring.

11.

Torque = 230 Nm at 60 rpm

DC source → DC motor → hoist shaft

150 V

10 A

2 minutes

↓

Lifts 3000 kg mass through a height of 5m.

- Work interaction for DC source = Power × time

$$= VIt$$

$$= 150 \times 10 \times (2 \times 60)$$

$$= 180 \text{ kJ}$$

- Work interaction for motor :-

DC source does work on DC motor,

thus, $W_{motor, source} = -180 \text{ kJ}$ = work done on DC motor by DC source.

Work done by DC motor on hoist

$= W_{hoist, motor}$ = Power × time = $T\omega t$ (torque × ω × time)

$$= 230 \times 2\pi \times 60 \frac{\text{radians}}{\text{minute}} \times 2 \text{ minute}$$

$$\approx 173.416 \text{ kJ}$$

∴ Total work interaction for motor.

$$= -180 + 173.416 \approx -6.584 \text{ kJ}$$

- Work interaction for hoist machine :-

DC motor does 173.416 kJ work on hoist.

Thus, $W_{hoist, motor} = -173.416 \text{ kJ}$

(negative because motor is doing work on hoist).

Work done by hoist in lifting 3000 kg mass,

$W_{mass, hoist} = 3000 \times 9.81 \times 5$

$= 147.15 \text{ kJ}$

$\therefore$ Work interaction for hoist machine $= -173.416 + 147.15$

$\approx -26.266 \text{ kJ}$

- Work interaction for mass

Hoist is doing work on mass.

$\therefore W_{mass, hoist} = -3000 \times 9.81 \times 5$

$= -147.15 \text{ kJ}$

(negative sign because hoist is doing work on mass).

It can be verified that

$$W_{DC} + W_{motor} + W_{hoist} + W_{mass} = 0$$